Electroweak theory

- Peskin 20.2
- Cheng-Li 11.2
- Schwartz 29

In this lecture we will describe the electroweak sector of the SM.

We just saw how SSB for SO(3) gauge theory leads to two massive vector bosons charged under the unbroken U(1).

This is compatible with muon decay, as it may be explained by the existence of heavy charged massive vectors.

The fact that the SU(2) ($\simeq$ SO(3) perturbatively) model contains charged vectors can be seen from the algebra:

Under SU(2), $\Omega = e^{i\alpha_a(x)\tau^a}$

$$W_\mu \to \Omega W_\mu \Omega^{-1} + i\Omega\partial_\mu\Omega^{-1}$$

Write the W_μ matrix as

$$W_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix}$$

Focus on transformations with $\alpha_1 = \alpha_2 = 0$.

Taking $\alpha_3 \simeq$ cnst,

$$W_\mu \longrightarrow e^{i\alpha\tau^3} W_\mu e^{-i\alpha\tau^3} \simeq W_\mu + i\alpha[\tau_3, W_\mu]$$

$$= W_\mu + i\alpha \frac{1}{2}\begin{pmatrix} 0 & W_1 - iW_2 \\ -(W_1 + iW_2) & 0 \end{pmatrix}$$

Some of the W's transform under this U(1) subgroup, since $U(1) \subset SU(2)$ τ^3 does not commute with all SU(2) generators.

$$W_\mu^3 \; : \quad \delta W_\mu^3 = 0 \quad \rightarrow \text{"neutral"}$$

$$W_\mu^+ \equiv \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}} \; : \quad \delta W^+ = \alpha W^+ \rightarrow \text{"charge +1"}$$

$$W_\mu^- \equiv \frac{W^1 + iW^2}{\sqrt{2}} \; : \quad \delta W^- = -\alpha W^- \rightarrow \text{"charge -1"}$$

One can check that the field strength

$$W_{\mu\nu} = W^a_{\mu\nu}\tau^a = \partial_\mu W_\nu - \partial_\nu W_\mu - i[\tau^a, \tau^b] W^a_\mu W^b_\nu$$

can be written in terms of a "photon" W^3_μ and two charged vectors with $D_\mu = \partial_\mu - i W^3_\mu$.

- Coupling to matter

We can introduce a Dirac fermion in the fund. of SU(2):

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad w/ \quad \psi \to e^{i\alpha_a \tau^a}\psi$$

The free lagrangian is

$$\mathcal{L} = i\bar{\psi}_1 \not\partial \psi_1 + i\bar{\psi}_2 \not\partial \psi_2 - m\bar{\psi}_1\psi_1 - m\bar{\psi}_2\psi_2$$

$$= i\bar{\psi}\not\partial\psi - m\bar{\psi}\psi$$

So $m_1 = m_2 = m$ is consistent with SU(2).

Fermion interactions fully specified by

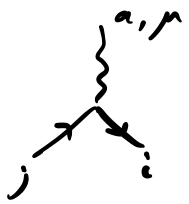
$$D_\mu\psi = \partial_\mu\psi - ig W^a_\mu \tau^a \psi$$

which leads to

$$\mathcal{L}_{int} = g \bar{\Psi} \gamma^\mu \tau^a \Psi W_\mu^a$$

$$= g (\tau^a)_i{}^j \bar{\Psi}^i \gamma^\mu \Psi_j W_\mu^a$$

leading to the vertex:



$$= i g (\tau^a)_i{}^j \gamma^\mu$$

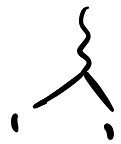
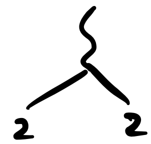
We can write it more explicitly in terms of $W^\pm$, W^3. Note that the two components have charge

$$Q(\Psi_1) = +1/2 \quad , \quad Q(\Psi_2) = -1/2$$

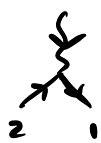
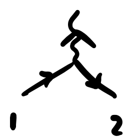
So, using
$$W_\mu = \frac{1}{2} \begin{pmatrix} W^3 & \sqrt{2} W^+ \\ \sqrt{2} W^- & -W^3 \end{pmatrix},$$

$$\mathcal{L}_{int} = \frac{g}{2} \bar{\Psi}_1 \not{W}^3 \Psi_1 - \frac{g}{2} \bar{\Psi}_2 \not{W}^3 \Psi_2 + \frac{g}{\sqrt{2}} \bar{\Psi}_1 \not{W}^+ \Psi_2 + \frac{g}{\sqrt{2}} \bar{\Psi}_2 \not{W}^- \Psi_1$$

So fermion interactions with W^3 are diagonal and proportional to fermion charge.

 $= i \frac{g}{2} \gamma^\mu$  $= - i \frac{g}{2} \gamma^\mu$

and interactions with $W^\pm$ are off-diagonal

 $= i \frac{g}{\sqrt{2}} \gamma^\mu$  $= i \frac{g}{\sqrt{2}} \gamma^\mu$

These are "charge current interactions" since they induce interactions among different components of the doublet.

However, this SU(2) model fails at the experimental level: Nature has neutral charged currents as well, and it is chiral.

- Higgs mechanism in the SM

Since we must add a vector current which is neutral, we need at least another generator.

The simplest option might be

$$SU(2) \times U(1)$$

which turns out to define the electroweak part of the SM:

$$\mathcal{G}_{SM} = SU(2)_L \times U(1)_Y$$

- L stands for "left", for reasons that will be evident.
- Y stands for "hypercharge". This name is historical, going back to the classification of hadrons.

Note that $U(1)_Y \neq U(1)_{em}$. In fact, the SM Higgs mechanism is defined by the symmetry breaking pattern

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}.$$

This is archieved by a doublet of scalars, the Higgs doublet

$$H = \begin{pmatrix} h_u \\ h_d \end{pmatrix} \begin{array}{l} \rightarrow \text{"up"} \\ \rightarrow \text{"down"} \end{array}$$

We assign it an hypercharge $Y(H) = +1/2$. This is purely conventional, as other assignments are equivalent up to a redefinition of the hypercharge coupling. (except $Y(H) = 0$! Can you see why?)

The $SU(2)_L \times U(1)_Y$ generators on H

are

$$\tau^a = \frac{\sigma^a}{2} \quad , \quad Y = \frac{1}{2}\mathbb{1}$$

with couplings g & g', respectively.

- We give H a potential

$$V(H) = \lambda\left(|H|^2 - \mu^2/2\lambda\right)^2$$

$$\Rightarrow \quad |\langle H\rangle| = \mu/\sqrt{2\lambda} \equiv \frac{v}{\sqrt{2}} \neq 0$$

We can expand the theory around

$$\langle H\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

We can see that

$$T_1\langle H\rangle = \frac{\sigma_1}{2}\langle H\rangle = \frac{1}{2}\begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$T_2\langle H\rangle = \frac{\sigma_2}{2}\langle H\rangle = -i\,\frac{1}{2}\begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$T_3\langle H\rangle = \frac{\sigma_3}{2}\langle H\rangle = -\frac{1}{2}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0$$

$$\frac{1}{2} Y \langle H \rangle = \frac{\mathbb{1}}{2} \langle H \rangle = \frac{1}{2} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0$$

So all T_i, Y are broken, but the combination

$$Q \equiv T^3 + Y = \frac{1}{2}(\tau^3 + \mathbb{1}) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

gives

$$Q \langle H \rangle = 0$$

So Q is unbroken and the corresponding gauge field is massless. The photon!

- Despite that the unbroken generator is now $T^3 + Y$, we can recycle many results from the $SU(2)$ case.

This is because $\mathcal{G}_{SM}$ is a direct product $SU(2) \times U(1)$, so

$$\Omega = \Omega_2 \times \Omega_Y$$

where

$$\Omega_2 = e^{i\alpha_a \tau^a} \quad , \quad \Omega_Y = e^{i\alpha_Y \frac{1}{2}\mathbb{1}} .$$

The gauge fields W_μ^a & B_μ (of $SU(2)_L$ & $U(1)_Y$, respectively) transform independently as

$$W_\mu \equiv W_\mu^a \tau^a \to \Omega_2 W_\mu \Omega_2^{-1} + i\Omega_2 \partial_\mu \Omega_2^{-1}$$

$$B_\mu \to B_\mu + \partial_\mu \alpha_Y$$

Therefore, a $U(1)_{em}$ transf.

$$\Omega_Q = e^{i\alpha_a \tau^a + i\alpha_Y \frac{1}{2}\mathbb{1}} \Big|_{\substack{\alpha_{1,2}=0 \\ \alpha_3 = \alpha_Y = \alpha_Q}}$$

$$= e^{i\alpha_Q \frac{1}{2}(\sigma^3 + \mathbb{1})}$$

$$= e^{i\alpha_Q Q}$$

The transf. acts on the W's like the SU(2) case.

We can write

$$W_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}\,W_\mu^+ \\ \sqrt{2}\,W_\mu^- & -W_\mu^3 \end{pmatrix}$$

so $W^\pm$ is a charged field, W^3 is neutral and under $U(1)_Q$

$$W_\mu^3 \to W_\mu^3 + \partial_\mu \alpha_Q$$

and the B_μ is also neutral and it transf. as

$$B_\mu \to B_\mu + \partial_\mu \alpha_Q$$

So the linear combination transforming under $U(1)_Q$ will be identified with the photon A_μ :

$$A_\mu \propto W_\mu^3 + B_\mu$$

while the $W_\mu^\pm$ <u>and</u> the orthogonal combination of W^3 and B_μ become massive and are identified with the W-bosons & the Z-boson

$$Z_\mu \propto W_\mu^3 - B_\mu$$

• We can check this by direct calculation.
Working in the unit. gauge, which in the SM corresponds to the choice

$$H = \begin{pmatrix} h_u \\ h_d \end{pmatrix} \xrightarrow{U.G.} \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$$

with h real.
Out of the 4 real d.o.f of H, 3 are set to zero in U.G., and reappear as the long. polarizations of $W^\pm$, Z.
The fourth, h, is the physical Higgs boson.

• The covariant derivative in U.G. is

$$D_\mu H = \partial_\mu H - i\left(W_\mu + \frac{1}{2} \mathbb{1} B_\mu \right)$$

$$= \begin{pmatrix} 0 \\ \frac{\partial_\mu h}{\sqrt{2}} \end{pmatrix} - i \, \frac{v+h}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\, W_\mu^+ \\ \underbrace{B_\mu - W_\mu^3}_{\hookrightarrow \; \propto Z_\mu .} \end{pmatrix}$$

By taking $|D_\mu H|^2$ we get a mass for W & Z.

Before computing the masses, recall that the gauge Lagrangian is

$$\mathcal{L} \supset -\frac{1}{2g^2}\,\mathrm{tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu}$$

Notice that the entire bosonic sector of the SM is controlled by 4 parameters:

$$g, g', \mu, \lambda \quad \underline{\text{or}} \quad g, g', v, \lambda$$

The gauge kinetic term is not canonical. We can rescale

$$W_\mu^{\pm,3} \to g\,W_\mu^{\pm,3} \quad \& \quad B_\mu \to g' B_\mu$$

This makes all couplings reappear in the interactions.

We can determine the normalization of Z_μ & A_μ by requiring that their canonical term is properly normalized.

Then

$$Z_\mu = c\,(g \underset{\substack{\downarrow \\ \text{can.}}}{W_\mu^3} - g' \underset{\substack{\downarrow \\ \text{can.}}}{B_\mu})$$

$$= \frac{1}{\sqrt{g^2+g'^2}}\,(g\,W_\mu^3 - g'\,B_\mu)$$

$$\equiv \cos\theta_W\, W_\mu^3 - \sin\theta_W\, B_\mu$$

with

$$\sin\theta_W \equiv s_W = \frac{g'}{\sqrt{g^2+g'^2}} \quad , \quad \cos\theta_W \equiv c_W = \frac{g}{\sqrt{g^2+g'^2}}$$

is the Weinberg angle.

The orthogonal combination is the photon field

$$A_\mu = c_W\, B_\mu + s_W\, W_\mu^3$$

- We can relate the $U(1)_Q$ coupling to g & g'.

With canonical norm, we have

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$$

under $U(1)$. Given that we have

$$W^3 \to W^3 + \frac{1}{g}\partial_\mu\alpha \quad , \quad B \to B + \frac{1}{g'}\partial_\mu\alpha$$

it implies

$$A_\mu = c_w B_\mu + s_w W^3_\mu \longrightarrow A_\mu + \left(\frac{c_w}{g'} + \frac{s_w}{g} \right) \partial_\mu \alpha$$

$$\rightarrow \frac{1}{e} = \frac{1}{\sqrt{g^2+g'^2}} \left(\frac{g}{g'} + \frac{g'}{g} \right) = \frac{\sqrt{g^2+g'^2}}{g g'} = \sqrt{\frac{1}{g^2} + \frac{1}{g'^2}}$$

or

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} \quad ; \quad e = \frac{g g'}{\sqrt{g^2+g'^2}}$$

The electric charge was known much before the SM, so this combination of param. is the first one being measured.

The measurement of the 4 parameters chronologically was

$e \leftrightarrow$ QED

$v \leftrightarrow$ Weak decays

$s_w \leftrightarrow$ ν-e scattering & Z-pole

$\lambda \leftrightarrow$ Higgs mass

with properly normalized fields, the Higgs kinetic term is

$$D_\mu H = \begin{pmatrix} 0 \\ \frac{\partial_\mu h}{\sqrt{2}} \end{pmatrix} - i \frac{v+h}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\, g\, W_\mu^+ \\ \sqrt{g^2+g'^2}\, Z_\mu \end{pmatrix}$$

$$\Rightarrow |D_\mu H|^2 = \frac{1}{2}(\partial h)^2 + \frac{(v+h)^2}{8}\left(2g^2|W|^2 + (g^2+g'^2) Z^2\right)$$

From which we read

$$m_W = \frac{1}{2} g v$$

$$m_Z = \frac{1}{2}\sqrt{g^2+g'^2}\, v = \frac{1}{2} g v \frac{1}{c_W} = \frac{m_W}{c_W}$$

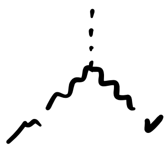
Notice the important relation

$$\rho = \frac{m_W^2}{m_Z^2\, c_W^2} = 1$$

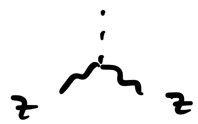
which holds at tree level in the SM.

This relation is a very important relation, that holds in the SM, but not necessarily if EWSB is driven by a sector more complex than a Higgs doublet. We shall come back to this point.

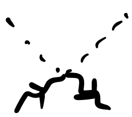
- From $(v+h)^2 = v^2 + 2vh + h^2$ we also get interactions among the Higgs & vectors



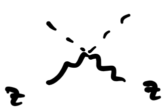
$$= \frac{i}{2} g^2 v \, \eta_{\mu\nu} = 2i \frac{m_W^2}{v} \eta_{\mu\nu}$$



$$= 2i \frac{m_Z^2}{v} \eta_{\mu\nu}$$



$$= i \frac{g^2}{2} \eta_{\mu\nu}$$



$$= \frac{i}{2} \frac{g^2}{c_W^2} \eta_{\mu\nu}$$

From the Higgs potential we get Higgs selfinterac.

$$V(H) \xrightarrow{\text{U.G.}} \lambda v^2 \rho^2 + \lambda v \rho^3 + \frac{\lambda}{4} \rho^4$$

$$\Rightarrow \quad m_H = \sqrt{2\lambda}\, v = \sqrt{2}\, \mu .$$

And Higgs self-interactions

 $= -6 i \lambda v$,  $= -6 i \lambda$

Finally, the gauge self-interactions can be obtained from the SU(2) discussion, as the B_μ field is abelian.

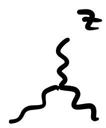
By inverting the Z_μ, A_μ relations,

$$W^3 = c_w Z + s_w A$$

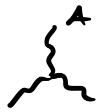
$$B = c_w A - s_w Z$$

So the SU(2) vertices become interactions for Z_μ & A_μ :

$\longrightarrow$



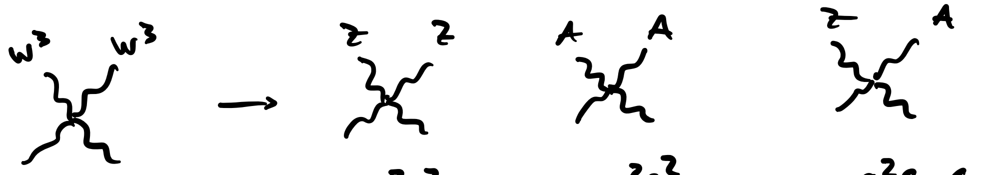
$= -i g c_w \left[(k_+ - k_-)^\rho \eta^{\mu\nu} + \dots \right]$



$= -i g s_w \left[\cdots \right]$

Notice that $g s_w = e$ the electric charge.
So the photon strength is fixed by the e.m. of $W^\pm$ bosons.

Similarly,



$\sim g^2 c_w^2$ $\qquad$ $\sim g^2 s_w^2$ $\qquad$ $\sim g^2 s_w c_w$

As a summary, at tree level we have

$$m_W = \frac{1}{2} g v = c_W m_Z$$

$$m_H = \sqrt{2\lambda}\, v$$

$$s_W = \frac{g'}{\sqrt{g^2+g'^2}} \quad ; \quad e = \frac{g g'}{\sqrt{g^2+g'^2}} = g s_W = g' c_W$$

Experimentally

$m_W \simeq 80\,\text{GeV}$ $\qquad$ $s_W^2 \simeq 0.23$

$m_Z \simeq 91\,\text{GeV}$ $\qquad$ $g \simeq 0.65$

$m_H \simeq 125\,\text{GeV}$ $\qquad$ $g' \simeq 0.35$

$v \simeq 246\,\text{GeV}$ $\qquad$ $\lambda \simeq 0.13$

$\mu \simeq 88\,\text{GeV}$